Structural Change in a Multi-Sector Model of Growth*

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Abstract

We study a multi-sector model of growth with differences in TFP growth rates across sectors and derive sufficient conditions for the coexistence of structural change, characterized by sectoral labor reallocation, and a balanced aggregate growth path, with all aggregates growing at the same rate. The conditions are weak restrictions on the utility and production functions: goods should be poor substitutes and the intertemporal elasticity of substitution should be one. We present evidence from US and UK sectors that is consistent with our conclusions and successfully calibrate the shift from agriculture to manufacturing and services in the United States.

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1 Introduction

This paper analyzes structural change and aggregate growth within a unified model. Structural change is the name normally given to the reallocation of factors across different sectors of the economy. The shifts between agriculture, manufacturing and services are the most commonly studied examples of structural change. We extend the one-sector optimizing model of economic growth with exogenous technological progress to many sectors, each having its own rate of total factor productivity growth. We show that the model is characterized by structural change and yet retains all the attractive features of the one-sector model, including, crucially, its steady-state growth equilibrium. The restrictions on functional forms needed to yield structural change and balanced growth are weak restrictions on functional forms that are frequently imposed by economists in related contexts.

Pioneering work on the connections between growth and structural change was done by Baumol (1967; Baumol et al., 1985). Baumol divided the economy into two sectors, a "progressive" one that uses capital and new technology and grows at some constant rate and a "stagnant" one that uses labor services as final output (as for example in the arts or the legal profession). He then claimed that because of factor mobility, the production costs and prices of the stagnant sector should rise indefinitely. Consequently, the stagnant sector should attract more labor to satisfy demand if demand is either income elastic or price inelastic, but should vanish otherwise. Baumol controversially also claimed that if the stagnant sector does not vanish the economy's growth rate will be on a declining trend, as more weight is shifted to the stagnant sectors.

We confirm Baumol's claim about structural change but also show that his conclusion, known as "Baumol's cost disease", was overly pessimistic. Although costs rise and resources shift into low-growth sectors during structural change, the growth rate of the aggregate economy is bounded from below by a positive rate that depends on the growth rate of Baumol's progressive sector and on the economy's savings rate.[1] Our economy satisfies Kaldor's stylized facts of constant rate of return to capital and constant rate of wage growth, even before it gets to the limiting state of no further structural change.

[1] Ironically, we get our result because of the inclusion of capital, a factor left out of the analysis by Baumol "for ease of exposition ... that is [in]essential to the argument". We show that the inclusion of capital is essential for the more optimistic growth results, though not for structural change.

We obtain our results by assuming that capital goods are supplied by only one sector, which we label manufacturing, and which produces also a consumption good. We show in an extension that they are consistent with the existence of many capital goods and intermediate goods. Production functions in our model are identical in all sectors except for their rates of TFP growth and each sector produces a differentiated good that enters a constant elasticity of substitution utility function. We show that a low (below one) elasticity of substitution across goods leads to shifts of employment shares to sectors with low TFP growth. In the limit the employment share used to produce consumption goods vanishes from all sectors except for the slowest-growing one, but the employment shares used to produce the capital good in manufacturing and any intermediuate goods in other sectors converge to non-trivial stationary values. We also show that if in addition the utility function has unit inter-temporal elasticity, the rate of return to capital is constant and a suitably-defined aggregate economy is on a steady-state growth path, which is obtained as the solution to two differential equations, one unstable in the control (aggregate consumption) and one stable in the state (the capital stock).

Our results contrast with the results of Echevarria (1997), who assumed non-homothetic preferences to derive structural change from different rates of sectoral TFP growth. In her economy balanced growth exists only in the limit, when preferences reduce to homotheticity with unit elasticity of substitution, and structural change ceases. In the transition the aggregate growth rate first rises and then falls, in contrast to ours, which is constant. Our results also contrast with the results of Kongsamut et al. (2001), who derive simultaneously constant aggregate growth and structural change. But they obtain their results by imposing a restriction that maps some of the parameters of their Stone-Geary utility function on to the parameters of the production functions, violating one of the most useful conventions of modern macroeconomics, the complete independence of preferences from technologies. Our restrictions are quantitative restrictions that maintain the independence of preferences and technologies.

In the empirical literature two competing explanations (which can coexist) have been put forward for structural change. Our explanation, which is sometimes termed "technological" because it attributes structural change to different rates of sectoral TFP growth, and a utility-based explanation, which requires different income elasticities for different goods and can yield structural change even with equal TFP growth in all sectors.[2] Kravis et al. (1983) present evidence that favours the technological explanation, at least when the comparison is between manufacturing and services. Two features of their data that are satisfied by the technological explanation are (a) relative prices have reflected differences in TFP growth rates and (b) real consumption shares have been fairly constant. Our model also has these implications when there is low substitutability across goods. We use multi-sector data for the United States and United Kingdom to show that changes in employment shares, prices and real consumption shares are consistent with our model's predictions. We also evaluate the model's performance in its explanation for the long-run shifts between agriculture, manufacturing and services. We show that although the model tracks the changes well, it predicts a slower decline of agriculture than is observed in the data. This leads us to conclude that although for manufacturing and services the technological explanation may be sufficient to explain structural change, the explanation for the fast decline of agriculture may require something additional, such as a below-unity income elasticity.

Section 2 describes our model of growth with many sectors and derives first the conditions for structural change and then the conditions for aggregate growth equilibrium. In section 5 we show some supporting evidence for our results by making use of US and UK sectoral data for 1970-1993. In section 6 we focus on the long-run structural change between manufacturing, agriculture and services and show both analytically and with computations the balanced growth path and the shift from agriculture to manufacturing and services and then from manufacturing to services, with shares matching reasonably well the shares observed in the United States. Finally, in section 7 we study two extensions of our benchmark model, one where there are many capital goods and one where consumption goods can also be used as intermediate inputs.

2 An economy with many sectors

The economy consists of an arbitrary number m of sectors. Sectors $i = 1, ..., m-1$ produce only consumption goods. The last sector, which is denoted by m and labeled manufacturing, produces both a final consumption good and the economy's capital

[2] Caselli and Coleman (2002) argue that another reason for structural change is exogenous changes in the supply of labor. They attribute the decline of agriculture to the increase in human capital, which made labor more productive in manufacturing and services than in agriculture.

stock. Manufacturing is the numeraire.[3]

Because markets are competitive and there are no externalities we derive the equilibrium as the solution to a social planning problem. The objective function is

$$U = \int_0^{\infty} e^{-\rho t} v\left(c_1, .., c_m\right) dt, \tag{1}$$

where $\rho > 0$, $c_i \geq 0$ are consumption levels and the instantaneous utility function $v\left(.\right)$ is concave and satisfies the Inada conditions. The constraints of the problem are as follows.

The labor force is exogenous and growing at rate ν and is allocated across sectors according to the employment shares n_i $\left(i = 1, ..., m\right)$. Therefore:

$$\sum_{i=1}^{m} n_i = 1; \qquad \sum_{i=1}^{m} n_i k_i = k. \tag{2}$$

where k_i denotes the capital-labor ratio in sector i and k denotes the aggregate capital-labor ratio.

All production in sectors $i = 1, ..., m-1$ is consumed but in sector m production may be either consumed or invested. Therefore:

$$c_i = F^i\left(n_i k_i, n_i\right) \qquad i = 1, ..., m-1 \tag{3}$$

$$\dot{k} = F^m(n_m k_m, n_m) - c_m - \left(\delta + \nu\right) k \tag{4}$$

where $F^i\left(.,.\right)$ is the production function of sector i and $\delta > 0$ is the depreciation rate.

The social planner chooses the allocation of factors n_i and k_i across the m sectors through a set of *static efficiency conditions,* and the allocation of output to consumption and capital through a *dynamic efficiency condition.* The static efficiency conditions are:

$$\frac{v_i}{v_m} = \frac{F_K^m}{F_K^i} = \frac{F_N^m}{F_N^i} \qquad \forall i. \tag{5}$$

and the dynamic efficiency condition is:

$$-\frac{\dot{v}_m}{v_m} = F_K^m - \left(\delta + \rho + \nu\right). \tag{6}$$

[3] The label manufacturing is used for convenience. Although in the standard industrial classifications our capital-goods producing sector belongs to manufacturing, some sectors classified as manufacturing in the data fall into the consumption category of our model. See below for more discussion of the empirical interpretation of our model.

where F_N^i and F_K^i are the marginal products of labor and capital in sector i. By the assumption that inter-sectoral capital and labor mobility are free, the rates of return to capital and labor are equalized across sectors.

Production functions are assumed to be Cobb-Douglas and in order to focus on the implications of different rates of TFP growth across sectors we assume that capital shares are constant across sectors:

$$F^i = A_i(n_i k_i)^\alpha n_i^{1-\alpha}; \quad \frac{\dot{A}_i}{A_i} = \gamma_i; \quad \alpha \in (0,1), \quad \forall i. \tag{7}$$

With these production functions, static efficiency and the resource constraints imply

$$k_i = k; \qquad p_i = \frac{v_i}{v_m} = \frac{A_m}{A_i}; \qquad \forall i, \tag{8}$$

where p_i is the price of good i in the decentralized economy (in terms of the price of the manufacturing good, i.e. $p_m \equiv 1$).

Utility functions are assumed to have constant elasticities both across goods and over time:

$$v(c_1, ..., c_m) = \frac{\phi(.)^{1-\theta} - 1}{1-\theta}; \quad \phi(.) = \left(\sum_{i=1}^{m} \omega_i c_i^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)} \tag{9}$$

where $\theta, \varepsilon > 0$, and $\omega_i > 0$ and $\sum_{i=1}^m \omega_i = 1$. Of course, if $\theta = 1$, $v(.) = \ln \phi(.)$ and if $\varepsilon = 1$, $\ln \phi(.) = \sum_{i=1}^m \omega_i \ln c_i$. The utility function is strictly concave and satisfies the Inada conditions. [4]

With the iso-elastic utility function the static efficiency conditions become:

$$\frac{p_i c_i}{c_m} = \left(\frac{\omega_i}{\omega_m}\right)^{\varepsilon} \left(\frac{A_m}{A_i}\right)^{1-\varepsilon} \equiv x_i \qquad \forall i \tag{10}$$

where x_i is the nominal consumption share of good i relative to the manufacturing good in the decentralized economy. By definition $x_m \equiv 1$, and so x_i is expressed in terms of the numeraire. We also define aggregate consumption and output in terms of the numeraire:

$$c \equiv \sum_{i=1}^{m} p_i c_i; \qquad y \equiv \sum_{i=1}^{m} p_i F^i \tag{11}$$

[4] Note that although $\phi(.)$ does not satisfy the Inada conditions, the utility function $v(.)$ does satisfy them.

Following these definitions, and using static efficiency, we can rewrite aggregate consumption and output as:

$$c = c_m X; \qquad y = A_m k^{\alpha} \tag{12}$$

where $X \equiv \sum_{i=1}^{m} x_i$. We note that the technology parameter for aggregate output is TFP in manufacturing and not an aggregate of all sectors' TFP.

3 Structural change

We define structural change as the state in which at least some of the labor shares change, i.e., $\dot{n}_i \neq 0$ for at least some i.

We derive in the Appendix (Lemma 6) the dynamic behavior of employment shares. For the consumption goods sectors, the employment shares satisfy:

$$n_i = \frac{x_i}{X}\left(\frac{c}{y}\right) \qquad i = 1, ..m-1, \tag{13}$$

and for the capital-producing sector:

$$n_m = \frac{x_m}{X}\left(\frac{c}{y}\right) + \left(1 - \frac{c}{y}\right). \tag{14}$$

The first term in the right side of (14) parallels the term in (13) and so represents the employment needed to satisfy the consumption demand for manufacturing goods. The second bracketed term is equal to the savings rate and represents the manufacturing employment needed to satisfy investment demand.

Condition (13) implies that employment in sector i relative to sector j depends only on the ratio x_i/x_j (for $i, j \neq m$) : $n_i/n_j = x_i/x_j$. By differentiation we obtain that the growth rate of relative employment depends only on the difference between the sectors' TFP growth rates and the elasticity of substitution between goods:

$$\frac{\dot{\overline{n_i/n_j}}}{n_i/n_j} = \frac{\dot{n}_i}{n_i} - \frac{\dot{n}_j}{n_j} = (1-\varepsilon)\left(\gamma_j - \gamma_i\right) \qquad \forall i, j \neq m. \tag{15}$$

But (8) implies that the growth rate of the relative price of good i is:

$$\frac{\dot{p}_i}{p_i} = \gamma_m - \gamma_i \qquad i = 1, ..., m-1 \tag{16}$$

and so,

$$\frac{\dot{\overline{n_i/n_j}}}{n_i/n_j} = (1-\varepsilon)\frac{\dot{\overline{p_i/p_j}}}{p_i/p_j} \qquad \forall i, j \neq m \tag{17}$$

Proposition 1 *Relative price changes depend only on differences in TFP growth rates; in sectors producing only consumption goods, changes in relative employment shares are proportional to changes in their relative prices, with the factor of proportionality monotonically falling in the price elasticity of demand.*[5]

The dynamics of the employment shares satisfy:

$$\frac{\dot{n}_i}{n_i} = \frac{\dot{\overline{c/y}}}{c/y} + (1-\varepsilon)(\bar{\gamma} - \gamma_i); \qquad i = 1, ...m-1 \tag{18}$$

$$\frac{\dot{n}_m}{n_m} = \left[\frac{\dot{\overline{c/y}}}{c/y} + (1-\varepsilon)(\bar{\gamma} - \gamma_m)\right] \frac{(c/y)(x_m/X)}{n_m} + \left(\frac{\dot{\overline{(1-c/y)}}}{(1-c/y)}\right)\left(\frac{1-c/y}{n_m}\right) \tag{19}$$

where $\bar{\gamma} \equiv \sum_{i=1}^{m} (x_i/X)\gamma_i$ is the weighted average of TFP growth rates.

Equation (18) gives the growth rate in the employment share of each consumption sector as a linear function of its own TFP growth rate. The intercept and slope of this function are common across sectors but although the slope is a constant, the intercept is in general a function of time. Manufacturing, however, does not conform to this rule, because its employment share is made up of two components, one for the production of the consumption good (which behaves similarly to the employment share of consumption sectors) and one for the production of investment goods.

With the help of these expressions we can now derive the properties of structural change. Consider first the case of equality in sectoral TFP growth rates, i.e., let $\gamma_i = \gamma_m \; \forall i$. Our economy in this case is one of balanced TFP growth, with relative prices remaining constant but with many differentiated goods. Because of the constancy of relative prices all consumption goods can be aggregated into one, so it is effectively a two-sector economy, one sector producing consumption goods and one producing capital goods. Structural change can still take place in this economy but only between the aggregate of the consumption sectors and the capital sector, and only if c/y changes over time. If c/y is increasing over time, the savings and investment rate are falling and labor is moving out of the manufacturing sector and into the consumption sectors. Conversely if c/y is falling over time. In both cases, however, the relative employment shares in consumption sectors are constant.

If c/y is constant over time, structural change requires $\varepsilon \neq 1$ and different rates of sectoral TFP growth rates. It follows immediately from (16), (18) and (19) that if $\dot{\overline{c/y}} = 0$, $\varepsilon = 1$ implies constant employment shares but changing prices. With

[5] All derivations and proofs, unless trivial, are collected in the Appendix.

constant employment shares faster growing sectors produce relatively more output over time. Price changes in this case are such that consumption demands exactly absorb all the output changes that are due to the TFP growth differentials. But if $\varepsilon \neq 1$, and although prices still change as before, consumption demands are either too inelastic (in the case $\varepsilon < 1$) to absorb all the output change, or are too elastic ($\varepsilon > 1$) so that the increase in TFP is not enough to give the required increase in output. So if $\varepsilon < 1$ employment has to move into the slow-growing sectors and if $\varepsilon > 1$ it has to move into the fast-growing sectors to satisfy consumption demands.

Proposition 2 *If $\gamma_i = \gamma_m$ $\forall i = 1, ..., m$, a necessary and sufficient condition for structural change is $\dot{c}/c \neq \dot{y}/y$. The structural change in this case is between the aggregate of consumption sectors and the manufacturing sector. If $\dot{c}/c = \dot{y}/y$, necessary and sufficient conditions for structural change are $\varepsilon \neq 1$ and $\exists i \in \{1, .., m-1\}$ s.t. $\gamma_i \neq \gamma_m$. The structural change in this case is between all sector pairs with different TFP growth rates.*

To obtain now the behavior of output and consumption shares we use the static efficiency results in (8) and (10) to derive:

$$\frac{p_i F^i}{\sum_{i=1}^{m} p_i F^i} = n_i; \qquad \frac{p_i c_i}{\sum_{i=1}^{m} p_i c_i} = \frac{x_i}{X}; \quad \forall i. \tag{20}$$

The nominal output shares are equal to the employment shares, so the results obtained for employment shares also hold for them. Relative employment shares but the relative real consumption shares satisfy:

$$\frac{\dot{c}_i}{c_i} - \frac{\dot{c}_j}{c_j} = \varepsilon \left(\gamma_i - \gamma_j \right); \qquad \forall i, j. \tag{21}$$

A comparison of (15) with (21) reveals that a small ε can reconcile the small changes in the relative real consumption shares with the large changes in both relative nominal consumption shares and relative employment shares which was reported by Kravis et al. (1983). This finding led the authors to conclude that the evidence favored a technological explanation for structural change. In section 5 we report multi-sector data that gives more support to the small ε.

4 Aggregate growth

Our results in the preceding section show that structural change partly depends on the behavior of the aggregate investment rate. The following Proposition defines and characterizes the equilibrium of the aggregate economy:

Proposition 3 *Given any initial k_0, the equilibrium of the aggregate economy is defined as a sequence $\{c_t, k_t\}_{t=0,1,...}$ that satisfies the following two dynamic equations:*

$$\frac{\dot{k}}{k} = A_m k^{\alpha-1} - \frac{c}{k} - (\delta + \nu), \tag{22}$$

$$\theta\frac{\dot{c}}{c} = (\theta - 1)(\gamma_m - \bar{\gamma}) + \alpha A_m k^{\alpha-1} - (\delta + \rho + \nu). \tag{23}$$

The key property of our equilibrium is that the contribution of each consumption sector i to aggregate equilibrium is through its weight x_i in the definition of the average TFP growth rate $\bar{\gamma}$. Note that because each x_i depends on the sector's relative TFP level, the weights here are functions of time.

We characterize the aggregate equilibrium by investigating whether there is an equilibrium path that satisfies Kaldor's fact of constant rate of return to capital. The rate of return to capital in each sector i is $\alpha p_i A_i k_i^{\alpha-1}$, so, given $p_m \equiv 1$ and $k_i = k$ $\forall i$, constant rate of return to capital requires that $A_m k^{\alpha-1}$ be constant, i.e., k should grow at rate $\gamma_m/(1-\alpha)$ and so, by (12), y/k must also be constant. But then the state equation (22) implies that c/k must also be constant, so in this steady state, if it exists, aggregate consumption grows at the same rate as the capital-labor ratio as well. We define this steady state as the balanced growth path.

We note that if all the γs are equal, relative prices are constant and the economy's average TFP growth rate is also the common γ. Our definition of aggregate consumption and output then correspond to the conventional definitions of real consumption and output, and our dynamic equations in Proposition 3 reduce to the conventional dynamic equations of the one-sector Ramsey economy. Given our results in Proposition 2, structural change takes place in the transition to the steady state of this economy, when c/y is changing, but not on the balanced growth path.

The more interesting case arises when at least some of the γs are different. In this case relative prices change and our definition of aggregate output and consumption are different from the conventional definitions, because they are deflated by the manufacturing price and not by an average of all prices. However, we can still talk of a balanced growth path defined as the state consistent with a constant rate of return to capital. We established in the preceding paragraph that on this path c/y is constant and so, by Proposition 2, structural change requires, in addition to the different γs, $\varepsilon \neq 1$. We now investigate whether such a balanced growth path exists.

By (23), a balanced growth path requires that the expression $(\theta-1)(\gamma_m-\bar{\gamma})$ be a constant. Let for now:

$$(\theta-1)(\gamma_m-\bar{\gamma}) \equiv \psi \qquad \text{constant.} \tag{24}$$

Define aggregate consumption and the aggregate capital-labor ratio in terms of efficiency units

$$c_e \equiv cA_m^{-1/(1-\alpha)}; \qquad k_e \equiv kA_m^{-1/(1-\alpha)}.$$

The dynamic equations become

$$\frac{\dot{c}_e}{c_e} = \frac{\alpha k_e^{\alpha-1}-(\delta+\nu+\rho)+\psi}{\theta}-\frac{\gamma_m}{1-\alpha} \tag{25}$$

$$\frac{\dot{k}_e}{k_e} = k_e^{\alpha-1}-\frac{c_e}{k_e}-\left(\frac{\gamma_m}{1-\alpha}+\delta+\nu\right). \tag{26}$$

Equations (25) and (26) parallel the two differential equations in the control and state of the one-sector model, making the aggregate equilibrium of our many-sector economy identical to the equilibrium of the one-sector Ramsey economy (when $\psi=0$) and trivially different from it otherwise. Both models have a saddle path equilibrium and stationary solutions $\left(\hat{c}_e,\hat{k}_e\right)$ that imply balanced growth in the three aggregates. As anticipated in the aggregate production function (12), a key result is that in our economy the rate of growth of our aggregates in the steady state is equal to the rate of growth of labor-augmenting technological progress in the sector that produces capital goods: the ratio of capital to employment in each sector and aggregate capital per worker grow at rate $\gamma_m/(1-\alpha)$. When nominal output is deflated by the price of manufacturing goods, output per worker and aggregate consumption per worker also grow at the same rate.

Proposition 2 and the results just derived give the important result:

Proposition 4 *Necessary and sufficient conditions for the existence of an aggregate balanced growth path with structural change are:*

$$\theta = 1, \tag{27}$$

$$\varepsilon \neq 1; \ \textit{and}\ \exists i\in\{1,..,n\}\ \textit{s.t.}\ \gamma_i\neq\gamma_m.$$

Under the conditions of Proposition 4, $\psi = 0$, and our aggregate economy becomes formally identical to the one-sector Ramsey economy. ψ is constant under two other (alternative) conditions, which give balanced aggregate growth, but as we argued in connection to Proposition 2, no structural change on the balanced growth path, $\gamma_i = \gamma_m \ \forall i$ and $\varepsilon = 1$.

Proposition 4 requires the utility function to be logarithmic in the consumption composite ϕ, which implies an intertemporal elasticity of substitution equal to one, but be non-logarithmic across goods, which implies non-unit price elasticities. A noteworthy implication of Proposition 4 is that balanced aggregate growth does not require constant rates of growth of TFP in any sector other than manufacturing. Because both capital and labor are perfectly mobile across sectors, changes in the TFP growth rates of consumption-producing sectors are reflected in immediate reallocations of capital and labor across the sectors (and in price changes), without effect on the aggregate growth path, which grows at rate γ_m irrespective of the values taken by the γ_i.

Proposition 4 confirms Baumol's (1967) claims about structural change. When demand is price inelastic, the sectors with the low productivity growth rate attract a bigger share of labor, despite the rise in their price. The lower the elasticity of demand, the less the fall in demand that accompanies the price rise, and so the bigger the shift in employment needed to maintain high relative consumption. But in contrast to Baumol's claims, the economy's growth rate is not on an indefinitely declining trend because of the existence of capital goods.

Next, we characterize the set of expanding sectors $(\dot{n}_i \geq 0)$, denoted E_t, and the set of contracting sectors $(\dot{n}_i \leq 0)$, denoted D_t, at any time t. We establish

Proposition 5 *Both in the balanced growth path and in the transition from a low initial capital stock, the set of expanding sectors is contracting over time and the set of contracting sectors is expanding over time:*

$$E_{t'} \subseteq E_t \text{ and } D_t \subseteq D_{t'} \qquad \forall t' > t$$

Asymptotically, the economy converges to a two-sector economy consisting of sector m and the sector that has the smallest (largest) TFP growth rate if and only if goods are poor (good) substitutes.

In order to give some intuition for the proof (which is in the Appendix), consider the dynamics of sectors on the balanced growth path. Along this path, the set of expanding and contracting sectors satisfy:

$$\begin{aligned} E_t &= \{i \in \{1,...,n,m\} : (1-\varepsilon)(\bar{\gamma} - \gamma_i) \geq 0\}; \\ D_t &= \{i \in \{1,...,n,m\} : (1-\varepsilon)(\bar{\gamma} - \gamma_i) \leq 0\}. \end{aligned} \tag{28}$$

If goods are poor substitutes ($\varepsilon < 1$), sector i expands if and only if its TFP growth rate is smaller than the weighted average of all sectors' TFP growth rates, and contracts if and only if its growth rate exceeds their weighted average. But if $\varepsilon < 1$, the weighted average $\bar{\gamma}$ is decreasing over time (see Lemma 7 in the Appendix). Therefore, the set of expanding sectors is shrinking over time, as more sectors' TFP growth rates exceed $\bar{\gamma}$.

If goods are good substitutes ($\varepsilon > 1$), sector i expands if and only if its TFP growth rate is greater than $\bar{\gamma}$, and contracts otherwise. But $\varepsilon > 1$ implies that $\bar{\gamma}$ is also increasing over time, so, as before, the set of expanding sectors is shrinking over time.

The asymptotic distribution of employment shares in the economy is

$$\begin{aligned} n_l^* &= \hat{c}_e \hat{k}_e^{-\alpha} = 1 - \alpha + \frac{\alpha\rho}{\delta + \nu + \rho + \gamma_m/(1-\alpha)} < 1 \\ n_m^* &= 1 - n_l^* \end{aligned} \tag{29}$$

where sector l denotes the sector with the smallest (largest) TFP growth rate if and only if goods are poor (good) substitutes. We note that n_l^* is equal to the ratio of aggregate consumption to output and so n_m^* is equal to the savings rate (equivalently, to the ratio of investment to output) along the balanced growth path. Denoting the savings rate by $\hat{\sigma}$ we obtain,

$$\hat{\sigma} = n_m^* = \alpha \left(\frac{\delta + \nu + \gamma_m/(1-\alpha)}{\delta + \nu + \rho + \gamma_m/(1-\alpha)} \right).$$

If there is no discounting $\rho = 0$, the employment share in the capital-producing sector is equal to α, the capital share in the economy as a whole. With $\rho > 0$ it is less than α. We can also see from (14) that $n_{mt} - n_m^* = n_l^*/X > 0$, i.e. the asymptotic employment share in manufacturing is smaller than its employment share along the balanced growth path at any point in time.

We conclude this section with an example of a simple economy, characterized by $\varepsilon < 1$, $\omega_i = \omega_m$, and $A_{i0} = A_{m0}\ \forall i$, i.e., one in which sectors differ only in their rates of TFP growth. Given these assumptions, the weights x_i equal 1 in all sectors at time 0. We rank the consumption sectors according to their TFP growth rate, letting sector n be the slowest growing sector. The weighted average of TFP growth rates at time 0 is the same as the mean TFP growth rate, $\bar{\gamma}_0 = \left(\frac{1}{m}\right)\sum_{i=1}^{m}\gamma_i$. Thus, initially sectors with a TFP growth rate below the mean are expanding, while sectors with a TFP growth rate above the mean are shrinking. The weight x_i is increasing if and only if $\gamma_i < \gamma_m$. Therefore, over time the weighted average $\bar{\gamma}$ is decreasing and, as claimed in proposition 5, the set of expanding sectors is shrinking and the set of contracting sectors is growing. Asymptotically, all sectors disappear sequentially according to their index until only sectors $m-1$ and m remain. As x_i equals 1 in all sectors at time 0, the initial employment shares are equal across consumption sectors, i.e. $n_{i0} = (1-\hat{\sigma})/m \; \forall i$. Over time, the consumption sectors that begin by losing employment contract until they disappear and those that begin by expanding eventually contract and disappear except for the $(m-1)$th sector, whose employment share converges to $1-\hat{\sigma}$ as $t \to \infty$. Employment in the only other remaining sector, manufacturing, converges to $\hat{\sigma}$.

5 Multi-sector evidence

Comprehensive multi-sector data that can be used to provide supporting evidence for our propositions exist since 1970. A full empirical test of our model will need to take into account barriers (institutional or otherwise) to factor mobility, which slow down the adjustment to our balanced growth equilibrium. We postpone this topic to future work.[6] Here we report some facts about structural change for the United States and the United Kingdom, as the two countries least likely to suffer from barriers to inter-sectoral allocations.

The three key implications of our model that we examine are summarized in equations (15), (16) and (17), which we re-write in the more convenient form:

$$\frac{\dot{n}_i}{n_i} - \frac{\dot{n}_j}{n_j} = -(1-\varepsilon)\left(\gamma_i - \gamma_j\right) \tag{30}$$

$$\frac{\dot{p}_i}{p_i} - \frac{\dot{p}_j}{p_j} = -(\gamma_i - \gamma_j) \tag{31}$$

$$\frac{\dot{n}_i}{n_i} - \frac{\dot{n}_j}{n_j} = (1-\varepsilon)\left(\frac{\dot{p}_i}{p_i} - \frac{\dot{p}_j}{p_j}\right) \tag{32}$$

[6] Nickell et al. (2004) recently estimated the pattern of "deindustrialization" across the OECD by using the same data set that we are using in this section and concluded that TFP differences are a major source of differences in the speed of deindustrialization observed in OECD countries.

Table 1: Sectors and Employment Shares

consumption goods			consumption + capital goods		
	empl. share			empl. share	
sector	US	UK	sector	US	UK
agriculture	3.9	3.3	mining	1.0	1.6
food	2.1	3.6	wood	1.5	-
textiles	2.7	3.9	paper	2.4	2.7
trade*	24.7	24.0	chemical	2.5	3.5
transport	5.3	7.7	non metallic	0.8	1.3
finance	5.4	10.4	metal	1.9	-
real est.+	8.9		machinery	10.1	15.1
services	17.4	9.4	other manuf.	0.5	0.5
			utilities	1.0	1.7
			construction	6.7	8.1

Notes. The time period for the US is 1970-93 and for the UK, 1970-90. * In the US, trade includes retail and wholesale trade; in the UK it includes in addition restaurants and hotels. + In the UK finance and real estate are grouped into one sector. There are no TFP data for the wood and metal sectors in the UK

Of course, these hold only for the $m-1$ consumption sectors and the third relation follows from the first two. However, because employment and prices at the sectoral level may be measured with less error than TFP, we report sectoral data for all three relations. If our model has predictive value the relations in (30)-(32) should hold, at least on average. The objective of the exercise in this section is to show whether these relations hold on average or not.

We use data from the OECD *International Sectoral Database* (ISDB), which covers the whole economy and is annual for the period 1970-1993. The ISDB has been merged with the *STAN Database for Industrial Analysis* and is no longer updated. However, ISDB contain data for sectoral TFP constructed by the OECD whereas STAN does not, so we chose to use the ISDB for the years that it is available. We extracted data for total employment, prices (obtained as the ratio of the sector's value added at current prices to value added at constant prices) and a TFP index for all SIC sectors. For each country we selected from the available sectors those that we considered to be predominantly consumption-goods sectors, by which we mean sectors whose final output is bought primarily by consumers, rather than businesses. These sectors are shown along with the other sectors in the database in Table 1. We excluded altogether from our analysis government services.

The relations in (30)-(32) hold both in and out of steady state but only when our assumptions of full information and full inter-sectoral labor and capital mobility are satisfied. For this reason they are more appropriate descriptions of long-run trends than year-to-year changes. We accounted for this by averaging the annual rates of growth of employment, prices and TFP for each sector over the entire sample and report results obtained with these averages. With 8 consumption sectors there are 28 sector pairs (in the UK there are 7 sectors and 21 pairs). Figure 1, panel (a), plots the differentials in the growth rates of the three variables against each other for the United States and figure 2, panel (a), repeats the same for the United Kingdom.

The three slopes in each diagram are as predicted by the model. The slope of the line in the price-TFP space is not significantly different from 1, in either the US or the UK. The employment-TFP slope gives $\varepsilon = 0.28$ for the United States and $\varepsilon = -0.34$ for the United Kingdom, but one that is not significantly different from zero. So the values of ε obtained from these plots are very small.[7] We argued that a small ε is crucial if our model is to explain the coexistence of large changes in employment shares with small changes in consumption shares and the multi-sector data appear to support both our technological reason for changes in employment shares and a small ε.

As a further test of the model, we repeat the same exercise for the non-consumption sectors. The model assumes the existence of only one capital-producing sector, so it is silent about the relations that should hold between capital-producing sub-sectors. We return to this when we consider an extension of our model with multiple capital producing sectors. Panels (b) in figures 1 and 2 give results comparable to those of panels (a) but for capital-producing sectors. The results contrast sharply with those found for the consumption sectors. For the United States none of the three diagrams shows a significant relation between the variables. For the United Kingdom, the points in the two diagrams with employment are again not showing significant relations and the only significant relation, between the growth differential of prices and TFP, gives a slope very close to 2, instead of the 1 obtained for the consumption sectors. There appears to be a sharp distinction in connection to changes in employment shares be-

[7] We estimated the two-equation system (30)-(31) under the assumption that each pair of sectors is subject to independent stochastic shocks, and imposed a unit coefficient on (31), which was easily accepted by the data. The estimates were $\varepsilon = 0.29$ (s.e.=0.19) for the United States and $\varepsilon = -0.01$ (s.e.=0.35) for the United Kingdom. We also estimated the slope of the line in employment-price space for comparison with the other slopes and obtained $\varepsilon = 0.36$ (s.e.=0.20) for the US and $\varepsilon = 0.27$ (s.e.=0.19) for the UK.

tween sectors that produce primarily goods for the household sector and those that produce capital goods, as emphasized by our model.[8]

6 Historical evidence

Long time series for an economy close to our frictionless economy exist for the United States since 1870. We now focus on the nature of long-run structural change predicted by the model, by computing the balanced growth path for an economy with three sectors, agriculture (sector a), services (sector s) and manufacturing (sector m), and compare the results with the US experience. But before doing that we compare our prediction of constant growth for the economy's aggregates in terms of the manufacturing numeraire and the aggregates normally reported by growth theorists, which use either fixed weights or chain-weighted series.

Our aggregate per capita income variable in (11) is, in nominal terms, $p_m y$, with the normalization $p_m \equiv 1$. So, if national statistics report real incomes deflated by some other implicit or explicit index $\tilde{p}$, reported real income is $p_m y/\tilde{p}$. The difference between our aggregate y and the reported one is the ratio of the price of our manufacturing goods to the deflator, $p_m/\tilde{p}$.When Kaldor and others looked at the long US time series and concluded that a constant rate of growth of per capita GDP is a "stylized fact" that could be imposed on aggregative models, they were looking at the rate of growth of $p_m y/\tilde{p}$.[9] In our model, the average relative manufacturing price does not grow at constant rate even on our balanced growth path because the relative sector shares that are used to calculate $\tilde{p}$ are changing during structural change. So it is not possible to have a precisely constant rate of growth of both our y and another aggregate $p_m y$ deflated by a weighted average of sector prices. But because sector shares do not change rapidly over time, at least visually, there is nothing to distinguish the "stylized fact" of constant growth in the chain-weighted (or fixed weights) per capita GDP and in our per capita output variable. The two series for the United States are

[8] Some of the sectors that we labelled manufacturing, e.g., paper, do not produce capital goods but we classified them as manufacturing because we think that a large fraction of their output is bought by businesses. We consider below an extension of our model with intermediate goods and show that although sectors that produce intermediate goods are closer to our consumption sectors than to manufacturing, structural change between them does not obey the simple relations in (30).

[9] More accurately, Kaldor was looking at a constant rate of return to capital, and others concluded that this requires a constant rate of growth of GDP. The motivation that led us to our definition of balanced growth was similar. See Kaldor (1961, p.178)

shown in Figure 3 since 1929, when the chain-weighted series becomes available. The rate of growth of the chain-weighted and our series are, respectively, 2.46 and 2.44 percent.

Turning now to the long-term shifts between agriculture, manufacturing and services, we note that if empirically the ranking of their TFP growth rates is such that $\gamma_a > \gamma_m > \gamma_s$, then the TFP growth rate for agriculture is always above the weighted average of TFP growth rates while the TFP growth rate for services is always below it, i.e. $\gamma_a > \bar{\gamma}_t > \gamma_s$ for all t. Therefore, the model predicts that if the three goods are poor substitutes, the agricultural employment share should decline indefinitely and the service sector employment share should rise. The manufacturing employment share may rise before it starts to decline if its TFP growth rate is lower than the initial economy-wide weighted average of TFP growth rates. But even if the share of manufacturing increases at first, eventually it should decline, as the weighted average of the TFP growth rates falls over time. Asymptotically, the three-sector economy converges to a two-sector economy with manufacturing and services only, with the employment share of manufacturing equal to the investment to output ratio along the balanced growth path.

From (13), the employment shares at any time t obey

$$\begin{aligned} n_{it} &= (1-\hat{\sigma})\frac{x_{it}}{X_t} \qquad i = a, s \\ n_{mt} &= 1 - n_{at} - n_{st}. \end{aligned} \tag{33}$$

Therefore, given any initial distribution of employment shares (n_{a0}, n_{s0}, n_{m0}), we have $x_{a0} = n_{a0}/\left(n_{m0} - \hat{\sigma}\right)$ and $x_{s0} = n_{s0}/\left(n_{m0} - \hat{\sigma}\right)$.[10] With information on the parameter ε and the TFP growth rates, the model generates the distribution of employment shares over time: given x_{i0}, ε and the $\gamma_i' s$, we derive x_{it}, then use (33) to derive n_{it}.

Consider now a plausible scenario for industrialized countries, an investment rate of 20 percent and an aggregate growth rate of 2 percent. Also, let initially half the labor force be in agriculture and the other half divided equally between manufacturing and services.[11] As demonstrated in Figure 4, the employment share of agriculture in

[10] This implies that n_m is first increasing if $\gamma_m < \left(n_{a0}\gamma_a + n_{s0}\gamma_s\right) / \left(n_{a0} + n_{s0}\right)$. So, initial employment shares and TFP growth rates are necessary and sufficient to determine whether n_m increases before it starts to decline.

[11] In other words, $\hat{\sigma} = 0.2$ and x_{a0} and x_{s0} are derived from initial employment shares. The 2 percent aggregate growth rate implies $\gamma_m = 0.012$ for $\alpha = 0.4$. Note that this implies that the labor productivity in the manufacturing sector is also growing at 2 percent. The rest of the parameters are $\gamma_a = 0.025$, $\gamma_s = 0.005$, $\varepsilon = 0.2$.

this scenario falls while the employment share of services rises, both monotonically. The employment share in manufacturing first rises slightly, then it flattens and finally it declines. The decline is more noticeable when the agricultural employment share becomes small.

The pattern implied by this scenario is a typical pattern of structural change observed in industrialized countries.[12]. The "shallow bell shape" for manufacturing that was found by Maddison (1980, p. 48) for each of the 16 OECD countries in his sample is a prediction that we believe is unique to our model. Figure 5 shows that the same patterns also hold when the employment shares are plotted against GDP for the 16 OECD countries in cross sections, using data from 1870 to 2001.[13]

To evaluate the quantitative implications of our model, we calibrate our balanced growth path to the US economy from 1869 to 1998. We describe how we conducted the calibration in the Appendix. Our model makes predictions about the aggregate economy, relative prices and employment shares. The strategy is to choose parameters to match the first two and let the model determine the dynamics of employment shares. In brief, we set $\hat{\sigma}$ to match the aggregate investment rate and γ_m to match the manufacturing growth rate, and (γ_s, γ_a) to match the average growth rate for the relative prices of agriculture and services in terms of manufacturing. We use values of ε that are consistent with the multi-sector evidence in Section 5. We then match the employment shares in 1869, and examine how the predictions of the model compare with the employment shares in the data. We exclude the government sector in the calculation of employment shares, as the services provided by government are not

[12]For example, Kuznets (1966) documented this pattern for 13 OECD countries and the USSR between 1800 and 1960. The 13 OECD countries are Australia, Belgium, Canada, Denmark, France, Great Britain, Italy, Japan, Netherlands, Norway, Sweden, Switzerland and US. Maddison (1980) and (1991) documented this pattern for 16 OECD countries from 1870 to 1987. The 16 countries include the 13 countries in Kuznets (1966), Austria, Finland and Germany.

[13]GDP per capita in 1990 international dollar are from Maddison (2001). Agriculture includes agriculture, forestry, and fishing; industry includes mining, manufacturing, electricity, gas and water supply, and construction. Services is a residual which includes government. The 16 OECD countries are the same as in Maddison (1980). The figure includes data for all countries in 1870, 1913, 1950, 1960, 1973, 1987 and 2001 with two exceptions: (1) only agriculture shares in Denmark, Japan and Switzerland for 1870, and (2)1913 only has France, Germany, Netherlands, Germany, UK and US.

priced optimally.[14] Thus, the baseline parameters are

$\hat{\sigma}$	γ_m	γ_a	γ_s	ε
0.2	0.013	0.023	0.003	0.3

Figure 6, panel (a), reports the results for our baseline parameters. Although the model captures the general features of the data, it fails to capture the full extent of the decline of agriculture. Figure 6, panel (b) allows for a lower elasticity of substitution, $\varepsilon = 0.1$, which improves the prediction for agriculture.[15] However, the model still predicts too high an employment share for agriculture, above 10 percent for 1990 to 1996, while it was smaller than 5 percent in the data. This suggests productivity growth alone is not sufficient to account for the decline in agriculture, but the model predicts well the allocations of non-agricultural employment between manufacturing and services. In the case of $\varepsilon = 0.3$ we overpredict agricultural employment by 16 percentage points and underpredict services and manufacturing employment by 14 and 2 points respectively. If we were to redistribute the 16 point surplus share from agriculture to manufacturing and services according to their existing share proportions, we obtain a share of manufacturing of 29 percent and a share of services of 68 percent, which compare favorably with their actual shares of 27 and 70 percent respectively.

A reason for the failure to match the decline of agriculture may be the unit income elasticity that we assumed. There seems to be a consensus in the literature that the income elasticity of demand for agricultural products is below unity, so a more appropriate utility function for agricultural goods may be one that includes a subsistence level, e.g., one that takes the form $v(c_a - \bar{c}_a, c_m, c_s)$, with $\bar{c}_a > 0$. A constant $\bar{c}_a$ would contribute to the fast decline of agricultural employment in the first stages of development, when most of consumption is accounted for by subsistence.[16]

[14] The employment shares are caculated using data from Historical Statistics for 1869-1959 and ISDB for 1960-1996.

[15] Note that this value is within one standard error of our estimates in Section 5.

[16] However, a subsistence level alone is still not capable of explaining the fast decline in agricultural employment. if the elasticity of substitution between the three goods is unity. It appears that less than unit elasticity of substitution is also needed, as in the models of Laitner (2000) and Gollin et al. (2002) where the elasticity of substitution is effectively 0 after a subsistence level of agricultural consumption has been satisfied. In contrast Caselli and Coleman (2002), assume a unit elasticity of substitution but match the fast decline of agricultural employment by assuming that the cost of moving out of agriculture fell because of the increase of education in rural areas.

7 Extensions: Many capital goods and intermediate inputs

Our baseline model has only one sector producing capital goods and no intermediate inputs. We now generalized it to allow more capital-producing sectors and also allow firms to buy the outputs of all sectors and use then as intermediate inputs. The motivation for many capital goods is obvious: more than one manufacturing sectors produce capital goods and we wish to study the implications of different TFP growth rates for each. The motivation for the introduction of intermediate inputs is that many of the sectors that would be classified as consumption sectors produce in fact for business. We already mentioned paper that sells, e.g., paper products to service industries. But even within the finance and service sector there are sub-sectors such as business services whose primary customers are other businesses.[17]

7.1 Many capital goods

We suppose that there are κ different capital-producing sectors each supplying the inputs into a production function G, which produces a capital aggregate that is used as an input in all production functions F^i. Thus, the model is the same as before, except that now the capital input k_i is not the output of a single sector but of the production function G. The Appendix derives the equilibrium for the case when G is a CES function with elasticity μ, i.e., when

$$G = \left[\sum_{j=1}^{\kappa} \xi_{m_j} \left(F^{m_j} \right)^{(\mu-1)/\mu} \right]^{\mu/(\mu-1)} \tag{34}$$

where $\mu > 0$, $\xi_{m_j} \geq 0$ and F^{m_j} is a Cobb-Douglas production function for each capital goods sector m_j. G is now our aggregate capital stock.

It follows immediately that the structural change results derived for the $m-1$ consumption sectors remain intact, as we have made no changes to that part of the model. But there are new results to derive concerning structural change within the capital-producing sectors. The relative employment shares across the capital-

[17] Kravis et al (1983) mention the importance of business services. For a more recent study that calculates the fraction of inputs into other production processes accounted for by services, using input-output tables, see Faini et al (2004).

producing sectors satisfy:

$$\begin{aligned}
\frac{n_{m_j}}{n_{m_i}} &= \left(\frac{\xi_{m_j}}{\xi_{m_i}}\right)^{\mu}\left(\frac{A_{m_i}}{A_{m_j}}\right)^{1-\mu}; \quad \forall i,j=1,..,\kappa & (35)\\
\frac{\dot{\overline{n_{m_j}/n_{m_i}}}}{n_{m_j}/n_{m_i}} &= (1-\mu)\left(\gamma_{m_i}-\gamma_{m_i}\right); \quad \forall i,j=1,..,\kappa &
\end{aligned}$$

If $\mu = 1$ (G is Cobb-Douglas), then the relative employment shares across capital-producing sectors remain constant over time. If $\mu > 1\,(< 1)$, then more productive capital-producing sectors have a higher (lower) employment share relative to the less productive capital-producing sector.

Comparing the new results to the results derived for consumption sectors in the baseline model, the A_m of the baseline model is replaced by $G_{m_j}A_{m_j}$, where G_{m_j} denotes the marginal product and A_{m_j} denotes TFP of capital good m_j. This term measures the rate of return to capital in the jth capital-producing sector, which is equal across all κ sectors because of free mobility of capital. Defining $A_m \equiv G_{m_1}A_{m_1}$ we derive the growth rate:

$$\gamma_m \equiv \frac{\dot{A}_m}{A_m} = \sum_{j=1}^{\kappa}\left(\frac{\xi_{m_j}}{\xi_{m_1}}\right)^{\mu}\left(\frac{A_{m_1}}{A_{m_j}}\right)^{1-\mu}\left(\gamma_{m_j}-\gamma_{m_1}\right)+\gamma_{m_1} \tag{36}$$

The dynamic equations for c and k are the same as the baseline model.

If TFP growth rates are equal across all capital-producing sectors, c and k grow at a common rate in the steady state. But then all capital producing sectors are identical and the model reduces to one with a single capital-producing sector.

If TFP growth rates are different across the capital-producing sectors and $\mu \neq 1$, then there is structural change within the capital-producing sectors along the transition to the asymptotic state. Asymptotically, only one capital-producing sector remains. In this state, c and k again grow at common rate, so there exists an asymptotic balanced growth path with only one capital-producing sector.

It follows that a necessary and sufficient condition for the coexistence of a balanced growth path and multiple capital-producing sectors with different TFP growth rates is $\mu = 1$. The aggregate growth rate in this case is $\gamma_m/(1-\alpha)$ and (36) implies $\gamma_m = \sum_{j=1}^{\kappa}\xi_{m_j}\gamma_{m_j}$. Using (35), the relative employment shares across capital-producing sectors are equal to their relative input shares in G. There is no structural change within the capital producing sectors induced by their TFP growth differences, their relative employment shares remaining constant independently of their TFP growth rates.

The extended model with $\varepsilon < 1$ and $\mu = 1$ predicts that along the balanced growth there is reallocation from high TFP growth consumption sectors to low TFP growth sectors but no relation between TFP growth rates and changes in employment shares across the capital-producing sectors. Figure 1 for the US economy confirms the model's prediction. For the UK, the data show a weak positive relationship between the relative employment shares across capital-producing sectors and their relative TFP growth rates but again not one that is statistical significant.

7.2 Intermediate Goods

Our second extension is more substantial. We allow all sectors to produce intermediate goods which can be used as an input in production by other sectors. The key difference between intermediate goods and capital goods is that capital goods are reusable while intermediate goods depreciate fully after one usage. As in the baseline model, sectors are of two types. The first type, which consists of sectors such as food and services, produces goods that are consumed either by households or firms. When goods are consumed by firms we call them intermediate inputs but refer to these sectors as consumption sectors for short. The second type of sector consists of sectors such as engineering and metals and produces goods that are used as capital. For generality's sake, we assume that the outputs of capital-producing sectors can also be processed into consumption goods or intermediate inputs. As before, we assume that there are $i = 1, ..., m-1$ consumption sectors and there is only one capital goods sector.

Formally, the production functions are modified as $F^i \equiv A_i n_i {k_i}^{\alpha} q_i^{\beta}$, $\forall i$, where q_i is the ratio of the intermediate goods to employment in sector i, β is its input share. When $\beta = 0$, we return to our baseline model. The output of consumption sector i is now $c_i + h_i$, where h_i is the output that is used as an input in an aggregate production function Φ that produces the intermediate good q. Manufacturing output can be consumed, c_m, used as an intermediate input, h_m, or used as capital, k. Restricting Φ to the CES class with elasticity $\eta > 0$, we show in the Appendix that a necessary and sufficient condition for a balanced growth path requires $\eta = 1$, i.e. Φ to be Cobb-Douglas. When this is true, all our results from the baseline model remain unaltered, except for the results for relative employment shares, (30), which require modification.

The aggregate equilibrium is similar to the baseline model with dynamic equations:

$$\frac{\dot{c}}{c} = \alpha A k^{(\alpha+\beta-1)/(1-\beta)} - (\delta+\rho+\nu), \tag{37}$$

$$\frac{\dot{k}}{k} = (1-\beta) A k^{(\alpha+\beta-1)/(1-\beta)} - \frac{c}{k} - (\delta+\nu) \tag{38}$$

where $A \equiv \left[A_m (\beta\Phi_m)^\beta\right]^{1/(1-\beta)}$ and Φ_m is the marginal product of the manufacturing good in Φ. The growth rate of A is constant and equal to $\gamma = \gamma_m + \left(\beta \sum_{i=1}^m \varphi_i \gamma_i\right) / (1-\beta)$, where φ_i is the input share of sector i in Φ. Therefore, we can define aggregate consumption and the aggregate capital-labor ratio in terms of efficiency units and obtain a balanced growth path where the common growth rate is $\left(\gamma_m + \beta \sum_{i=1}^m \varphi_i \gamma_i\right) / (1-\alpha-\beta)$. Recall the aggregate growth rate in the baseline model depended only on the TFP growth rate in manufacturing. In the extended model with intermediate goods, the TFP growth rates in all sectors contribute to aggregate growth.

The employment shares are now:

$$n_i = \left(\frac{c}{y}\right)\left(\frac{x_i}{X}\right) + \varphi_i\beta; \qquad i = 1, ..., m-1 \tag{39}$$

$$n_m = \left[\left(\frac{c}{y}\right)\left(\frac{x_m}{X}\right) + \varphi_m\beta\right] + \left[1 - \beta - \frac{c}{y}\right] \tag{40}$$

which are intuitive compared to (13) and (14). For the consumption sectors, the extra term in (39) is the term $\varphi_i\beta$ which captures the employment required for producing intermediate goods. This is because the model implies φ_i is the share of sector i's output for intermediate purpose and β is the aggregate intermediate to aggregate output ratio. For the manufacturing sector, the terms in the first bracket parallel that of consumption sectors. The second term captures the employment for investment purpose. The relative employment shares across consumption sectors are no longer equal to x_i/x_j (as in the baseline model) because of the presence of intermediate goods. Therefore, Proposition 1 only holds for relative prices, but not for relative employment. The modification, however, is straightforward because $\varphi_i\beta$ is constant, and the results about the direction of structural change hold as in the baseline model. What does not hold now is the strong prediction about relative employment shares illustrated in Figures 1 and 2, although again, these relations should hold approximately when the fraction of a sector's output bought by business users is small. The constant term in the employment shares also affects the asymptotic results in Proposition 5. Asymptotically, the employment share used for the production of consumption goods still vanishes in all sectors except for the slowest growing one, but the employment share used to produce intermediate goods, $\varphi_i \beta$, survives in all sectors.

8 Conclusion

Economic growth takes place at uneven rates across different sectors of the economy. This paper had two objectives related to this fact, (a) to derive the implications of uneven sectoral growth for structural change, the shifts in sectoral employment shares that take place over long periods of time, and (b) to show that even taking into account the different sectoral rates of productivity growth there can still be balanced growth in the aggregate economy. We have shown that balanced growth requires some quantitative restrictions on parameters, the most important being a logarithmic intertemporal utility function. Predicted sectoral change that is consistent with the facts requires in addition low substitutability between the final goods produced by each sector. We have shown that underlying the balanced aggregate growth there is a shift of employment away from sectors with high rate of technological progress towards sectors with low growth, and eventually, in the limit, only two sectors survive, the sector producing capital goods and the sector with the lowest rate of productivity growth.

An examination of the facts for the United States and the United Kingdom has shown that our predictions are consistent with the facts, and that focusing on uneven sectoral growth and abstracting from all other causes of structural change (such as different capital intensities and non-unit income elasticities) can explain a large fraction of the observed employment shifts. More specifically, it can explain large parts of the shift of employment from agriculture to manufacturing and services and subsequently from manufacturing to services, albeit at a lower rate than is observed in the data. Of course, enriching the model with different capital intensities and non-unit income elasticities may improve the predictions. Future empirical work also needs to deal with intermediate goods and frictions in factor mobility. We have shown in an extension that intermediate goods alter some of our conclusions although not the important ones about structural change.

Finally, our model has implications for studies that take structural change as a fact and calculate its contribution to overall growth (Broadberry, 1998, Temple, 2001). For example, Broadberry and others calculate an economy's growth rate under the counterfactual of no structural change. They then attribute the difference between the actual growth rate and their hypothetical rate to structural change. Their approach has parallels with Baumol's approach (see also Triplett and Bosworth, 2003) who claim that the shift of weight in the aggregate economy to low growth service sectors should reduce the overall growth rate of the economy. Our model shows that structural change is a necessary part of aggregate growth and may shed new light on how to design accounting exercises of this kind. Temple (2001) uses growth accounting to calculate the contribution of structural change to overall growth, on the premise that labor moves from sectors which have low marginal product of labor to sectors that have high marginal product. But our analysis shows that labor moves because technological progress raises the marginal product of labor in the origin sectors and the prices of sectors in the receiving sectors. This reallocation mechanism, which is quite distinct from the one that he assumed and may be additional to it, sheds new light on the kind of decomposition that he does. Ultimately, the objective of these exercises is to understand the causes of growth and our approach suggests that structural change is an outcome for growth, not a cause.

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9 Appendix

Lemma 6 *The employment shares satisfy:*

$$\begin{aligned} n_i &= \left(\frac{x_i}{X}\right)\left(\frac{c}{y}\right), \qquad \frac{\dot{n}_i}{n_i} = \frac{\dot{\overline{c/y}}}{c/y} + (1-\varepsilon)(\bar{\gamma}-\gamma_i); \qquad i = 1, ...m-1, \\ n_m &= \left(\frac{c/y}{X}\right) + 1 - \frac{c}{y}, \quad \dot{n}_m = \left[\frac{\dot{\overline{c/y}}}{c/y} + (1-\varepsilon)(\bar{\gamma}-\gamma_m)\right]\left(\frac{c/y}{X}\right) - \dot{\overline{c/y}} \end{aligned}$$

where $\bar{\gamma} \equiv \sum_{i=1}^m (x_i/X)\gamma_i$ *is the weighted average of TFP growth rates.*

Proof. n_i follows from substituting F^i into (10), and n_m is derived from (2). Given $\dot{x}_i/x_i = (1-\varepsilon)(\gamma_m - \gamma_i)$ and $\dot{X} = \sum_{i=1}^m \dot{x}_i = (1-\varepsilon)(\gamma_m - \bar{\gamma})X$, we have

$$\frac{\dot{n}_i}{n_i} = \frac{\dot{\overline{c/y}}}{c/y} + \frac{\dot{\overline{x_i/X}}}{x_i/X} = \frac{\dot{\overline{c/y}}}{c/y} + (1-\varepsilon)(\bar{\gamma}-\gamma_i) \quad i = 1, ..m-1$$

and by (2),

$$\begin{aligned} \dot{n}_m &= -\sum_{i=1}^{m-1}\dot{n}_i = -\frac{\dot{\overline{c/y}}}{c/y}(1-n_m) - (1-\varepsilon)\left(\frac{c/y}{X}\right)\sum_{i=1}^{m-1} x_i(\bar{\gamma}-\gamma_i) \\ &= \frac{\dot{\overline{c/y}}}{c/y}\left(\frac{c/y}{X} - \frac{c}{y}\right) + (1-\varepsilon)\left(\frac{c/y}{X}\right)(\bar{\gamma}-\gamma_m) \\ &= \left[\frac{\dot{\overline{c/y}}}{c/y} + (1-\varepsilon)(\bar{\gamma}-\gamma_m)\right]\left(\frac{c/y}{X}\right) - \dot{\overline{c/y}}. \end{aligned}$$

■

Proof of Proposition 3. Use (2) and (8) to rewrite (4) as:

$$\dot{k}/k = A_m k^{\alpha-1}(1-\textstyle\sum_{i=1}^{m-1} n_i) - c_m/k - (\delta+\nu).$$

But $p_i = A_m/A_i$ and by definition of c, it is equivalent to:

$$\dot{k}/k = A_m k^{\alpha-1} - c/k - (\delta+\nu).$$

Next, ϕ is homogenous of degree one: $\phi = \sum_{i=1}^m \phi_i c_i = \sum_{i=1}^m p_i c_i \phi_m = \phi_m c$. But $\phi_m = \omega_m(\phi/c_m)^{1/\varepsilon}$ and $c = c_m X$, thus $\phi_m = \omega_m^{\varepsilon/(\varepsilon-1)} X^{1/(\varepsilon-1)}$ and $v_m = \phi^{-\theta}\phi_m = \left(\omega_m^{\varepsilon/(\varepsilon-1)} X^{1/(\varepsilon-1)}\right)^{1-\theta} c^{-\theta}$, so (6) becomes

$$\theta\dot{c}/c = (\theta-1)(\gamma_m - \bar{\gamma}) + \alpha A_m k^{\alpha-1} - (\delta+\rho+\nu).$$

Lemma 7 $d\bar{\gamma}/dt \lesseqgtr 0 \Leftrightarrow \varepsilon \lesseqgtr 1$.

Proof. Totally differentiating $\bar{\gamma}$ as defined in Proposition 3 we obtain

$$\begin{aligned} d\bar{\gamma}/dt &= \textstyle\sum_{i=1}^{m} (x_i/X)\, \gamma_i\, (\dot{x}_i/x_i - \sum_{i=1}^{m} \dot{x}_j/X) \\ &= (1-\varepsilon) \textstyle\sum_{i=1}^{m} (x_i/X)\, \gamma_i \left(\gamma_m - \gamma_i - \sum_{i=1}^{m} (x_i/X)(\gamma_m - \gamma_j)\right) \\ &= (1-\varepsilon)\left(\bar{\gamma}^2 - \textstyle\sum_{i=1}^{m} (x_i/X)\, \gamma_i^2\right) = -(1-\varepsilon) \textstyle\sum_{i=1}^{m} (x_i/X)(\gamma_i - \bar{\gamma})^2 \end{aligned}$$

Since the summation term is always positive the result follows. ■

Proof of Proposition 5

Lemma 8 *Along balanced growth path, if* $\varepsilon \lesseqgtr 1$, $\forall i = 1, .., m-1$, n_i *is non-monotonic if and only if* $\bar{\gamma}_0 \gtreqless \gamma_i$. *The non-monotonic* n_i *first increases at a decreasing rate for* $t < t_i$, *then decreases and converge to constant* n_i^* *asymptotically, where* t_i *is such that* $\bar{\gamma}_{t_i} = \gamma_i$. *The monotonic* n_i *is decreasing and converges to zero asymptotically.*

Proof. $\forall i = 1, .., m-1$, Lemma 6 implies that along balanced growth path, $\dot{n}_i/n_i = (1-\varepsilon)(\bar{\gamma} - \gamma_i) > 0 \Leftrightarrow \bar{\gamma}_t > \gamma_i$. Lemma 7 implies n_i eventually decreases. Therefore, n_i is non-monotonic if and only if $\bar{\gamma}_0 > \gamma_i$. ■

Corollary 9 *If* $\varepsilon < 1$, $t_s \to \infty$ *where* s *is such that* $\gamma_s = \min\{\gamma_i\}_{i=1,.,m}$. *If* $\varepsilon > 1$, $t_f \to \infty$ *where* f *is such that* $\gamma_f = \max\{\gamma_i\}_{i=1,..,m.}$

To establish now the claims in Proposition 5, assume, without loss of generality, $\varepsilon < 1$, $\gamma_1 > . > \gamma_{m-1}$ and $\gamma_m < \gamma_h = \bar{\gamma}_0$ where $1 < h < m-1$. Then, Lemma 8 implies $t_i = 0\ \forall i \leq h$, and $i \in E_0\ \forall i \geq h$, moreover, $E_{t_{h+1}} \cup \{h+1\} = E_0$ and $D_{t_h+1} = D_0 \cup \{h+1\}$, thus $E_{t_{h+1}} \subseteq E_0$ and $D_0 \subseteq D_{t_{h+1}}$.The result follows for any arbitrary $t > 0$. Next, we prove that the economy converges to a two-sector economy. Without loss of generality, consider $\varepsilon < 1$. Given $X/x_i = \sum_{i=1}^{m} (\omega_j/\omega_i)^{\varepsilon} (A_i/A_j)^{1-\varepsilon}$, and $A_i/A_j \to 0 \Leftrightarrow \gamma_i < \gamma_j$, we have $X/x_i \to 1 \Leftrightarrow \gamma_i = \min\{\gamma_j\}_{j=1,.,m}$. Therefore, asymptotically, $n_l^* = \hat{c}_e \hat{k}_e^{-\alpha}$ and $n_m^* = 1 - n_l^*$, where $\gamma_l = \min\{\gamma_i\}_{i=1,.,m}$.

We now prove these results hold also in transition to the steady state from any small k_0. Let $z \equiv c_e/k_e$, (25) and (26) (with $\psi = 0$ and $\theta = 1$) imply:

$$\dot{z}/z = (\alpha - 1)\, k_e^{\alpha-1} + z - \rho, \qquad \dot{k}_e/k_e = k_e^{\alpha-1} - z - [\gamma_m/(1-\alpha) + \delta + \nu].$$

A phase diagram can be drawn with $\dot{z} < 0$ along the transition. For c/y, we have:

$$\frac{\dot{\overline{c/y}}}{c/y} = \frac{\dot{c}_e}{c_e} - \alpha \frac{\dot{k}_e}{k_e} = \alpha z - \rho - (1-\alpha)\left(\frac{\gamma_m}{1-\alpha} + \delta + \nu\right).$$

Since $\dot{\overline{c/y}} = 0$ in the steady state but $\dot{z} < 0$ in the transition, thus $\dot{\overline{c/y}} > 0$ and $\ddot{\overline{c/y}} < 0$ along the transition. Also, $\forall t, \forall i = 1, .., m-1$, we have:

$$\dot{n}_i/n_i = \alpha z - \rho - (1-\alpha)\left[\gamma_m/(1-\alpha) + \delta + \nu\right] + (1-\varepsilon)(\bar{\gamma} - \gamma_i),$$

which decreases along the transition given lemma 7 and $\dot{z} < 0$. Thus, starting from any small k_0, if $i \in E_0$ then $\dot{n}_i > 0$, $\ddot{n}_i < 0$, and if $i \neq l$, $i \in E_t$ $\forall t < t_i$, and $i \in D_t$ $\forall t \geq t_i$, where t_i is defined in Lemma 8. If $i \in D_0$, then $i \in D_t$ $\forall t$. Therefore, Lemma 8 holds along the transition.

Calibration of the baseline model The parameters are the preference parameters $(\omega_a, \omega_m, \omega_s, \rho)$, the technology parameters $(\gamma_m, \gamma_s, \gamma_a, A_{a0}, A_{m0}, A_{s0,}\alpha, \delta)$ and the labor force growth rate ν. Given $\gamma_m/(1-\alpha)$, the roles of the parameters (δ, ν, ρ) are summarized through $\hat{\sigma}$ while the roles of $(\omega_a, \omega_m, \omega_s)$ and (A_{a0}, A_{m0}, A_{s0}) are summarized through the initial weights (x_{a0}, x_{s0}) which are set to match the employment shares in 1870. Therefore, there are only 5 parameters, $(\hat{\sigma}, \gamma_m, \gamma_s, \gamma_a, \alpha)$ to calibrate.

$(\hat{\sigma})$: We set $\hat{\sigma}$ to 0.2 which is about the average investment rate during 1870-2000. The investment rate is very much constant during 1870-2000 except the great depression and the war periods. The evidence can be found in Maddison (1992) for the period 1870-1988, Bureau of Economic Analysis for 1929-2000 and also in the Penn World Tables for the period 1950-2000.

To determine $(\alpha, \gamma_a, \gamma_m, \gamma_s)$, we use data from two main sources: (1) *Historical Statistic of the United States: Colonial Times to 1970*, Part 1 and 2: for the sectoral employment (series F250-258), relative prices (series E17, E23-25, E42, E52-E53) and index of manufacturing production (series P13-17), and (2) *Economic Report of the President*: for the relative prices and index of manufacturing production.

(γ_m, α) : The model implies that the aggregate growth rate is the same as the growth rate of labor productivity in manufacturing, and both are equal to $\gamma_m/(1-\alpha)$. The average annual growth rate of labor productivity in manufacturing is 2.2 percent between 1869 and 1998, which is consistent with the finding for the aggregate growth rate. The role of α in the quantitative analysis is through its influence on the implied γ_m, which is between 0.013 ($\alpha = 0.4$) and 0.014 ($\alpha = 1/3$). The results are robust to this range. We only report results with $\gamma_m = 0.013$.

(γ_a, γ_s) : The model implies the growth rate of relative price p_i is equal to $\gamma_m - \gamma_i$. The price data for agriculture and manufacturing start from 1869. However, the price data for services start in 1929. The average annual growth rate for the relative price of services in terms of manufacturing is -0.01 for the period 1929-1998, which implies $\gamma_s = 0.003$. The average annual growth rate of agriculture relative to manufacturing price for 1869-1998 is -0.003, which implies $\gamma_a = 0.016$. However, if we use the same period as for the service sector, i.e. for the period 1929-1998, the annual growth rate becomes -0.01, which implies $\gamma_a = 0.023$. What is important for the shift between the agriculture and service sector is the difference $\gamma_a - \gamma_s$, so we use the same period for both prices series, $\gamma_a = 0.023$ and $\gamma_s = 0.003$.

(ε) : Ideally, we want an estimate for the elasticity of substitution for the period 1869-1998. Without this measure, we use $\varepsilon = 0.3$, which is for the period 1970-1993. But we also report results for a lower value of $\varepsilon = 0.1$.

Many capital-producing sectors $\forall j = 1, ..\kappa$, $F^{m_j} \equiv A_{m_j} n_{m_j} k_{m_j}^{\alpha}$, which together produce good m through $G = \left[\sum_{j=1}^{\kappa} \xi_{m_j} \left(F^{m_j}\right)^{(\mu-1)/\mu}\right]^{\mu/(\mu-1)}$, $\xi_{m_j}, \mu > 0$, and $\sum_{j=1}^{\kappa} \xi_{m_j} = 1$. The planner's problem is similar to before with $\dot{k} = G - c_m - (\delta + \nu) k$ replacing (4), and $\left(k_{m_j}, n_{m_j}\right)_{j=1,.,\kappa}$ as additional control variables.

The static efficiency conditions are $F_K^i / F_N^i = F_K^{m_j} / F_N^{m_j}$, $\forall i = 1, ..m-1$, $\forall j = 1, ., \kappa$, so $k_i = k_{m_j} = k$. Also $G_{m_j} / G_{m_i} = F_K^{m_i} / F_K^{m_j} = A_{m_i} / A_{m_j}$, $\forall i, j = 1, ..\kappa$, which implies $n_{m_j} / n_{m_i} = \left(\xi_{m_j} / \xi_{m_i}\right)^{\mu} \left(A_{m_i} / A_{m_j}\right)^{1-\mu}$ and grows at $(1-\mu)\left(\gamma_{m_i} - \gamma_{m_i}\right)$. Let $n_m \equiv \sum_{j=1}^{\kappa} n_{m_j}$, we have $n_m = n_{m_1} \sum_{j=1}^{\kappa} \left(\xi_{m_j} / \xi_{m_1}\right)^{\mu} \left(A_{m_1} / A_{m_j}\right)^{1-\mu}$. Next, $\forall i = 1, ..m-1$, $p_i \equiv v_i / v_m = A_m / A_i$, where $A_m \equiv G_{m_1} A_{m_1}$. Thus, n_i / n_j and p_i / p_i are the same as the baseline model. For the aggregate equilibrium, note $G = \sum_{j=1}^{\kappa} F^{m_j} G_{m_j} = A_m k^{\alpha} n_m$, so $\dot{c}/c$ and $\dot{k}/k$ become the same as the baseline model. Thus, same equilibrium if $\dot{A}_m / A_m$ is constant. Note $G_{m_1} = \xi_{m_1} \left(G / F^{m_j}\right)^{1/\mu} = \xi_{m_1} \left(G_{m_1} n_m / n_{m_1}\right)^{1/\mu} = G_{m_1}^{1/\mu} \left[\sum_{j=1}^{\kappa} \left(\xi_{m_j} / \xi_{m_1}\right)^{\mu} \left(A_{m_1} / A_{m_j}\right)^{1-\mu}\right]^{1/\mu}$, so

$$
\begin{aligned}
G_{m_1} &= \xi_{m_1}^{\frac{\mu}{\mu-1}} \left[\sum_{j=1}^{\kappa} \left(\xi_{m_j} / \xi_{m_1}\right)^{\mu} \left(A_{m_1} / A_{m_j}\right)^{1-\mu}\right]^{\frac{\mu}{\mu-1}} \\
&\implies \dot{G}_{m_1} / G_{m_1} = \sum_{j=1}^{\kappa} \left(\xi_{m_j} / \xi_{m_1}\right)^{\mu} \left(A_{m_1} / A_{m_j}\right)^{1-\mu} \left(\gamma_{m_j} - \gamma_{m_1}\right) \\
&\implies \gamma_m \equiv \dot{A}_m / A_m = \sum_{j=1}^{\kappa} \left(\xi_{m_j} / \xi_{m_1}\right)^{\mu} \left(A_{m_1} / A_{m_j}\right)^{1-\mu} \left(\gamma_{m_j} - \gamma_{m_1}\right) + \gamma_{m_1}
\end{aligned}
$$

which is constant if $(1-\mu) \sum_{j=1}^{\kappa} \left(\xi_{m_j} / \xi_{m_1}\right)^{\mu} \left(A_{m_1} / A_{m_j}\right)^{1-\mu} \left(\gamma_{m_j} - \gamma_{m_1}\right)^2 = 0$, i.e. if (1) $\gamma_{m_i} = \gamma_{m_j}$ $\forall i, j = 1, ..\kappa$ or (2) $\mu = 1$. If (1) is true, it reduces to a model with only one capital-producing sector. Thus, coexistence of multiple capital-producing sectors and balanced growth path requires (2), i.e., $G = \prod_{j=1}^{\kappa} \left(F^{m_j}\right)^{\xi_j}$, which implies

$A_m = G_{m_1} A_{m_1} = A_{m_1} \xi_{m_1} G / F^{m_1} = \prod_{j=1}^{\kappa} \left(\xi_{m_j} A_{m_j} \right)^{\xi_j}$ and $\gamma_m = \sum_{j=1}^{\kappa} \xi_{m_j} \gamma_{m_j}$.

Intermediate goods Introducing intermediate goods into production functions, $F^i \equiv A_i n_i k_i{}^{\alpha} q_i^{\beta}$, $\forall i$, $\alpha, \beta \in (0,1)$ and $\alpha + \beta < 1$. For $i = 1, .., m-1$, F^i is either bought by consumers (c_i) or by business (h_i). But F^m can also be used as investment. Intermediate goods are produced by $\Phi(h_1, .., h_m)$, which satisfy $\Phi_i > 0, \Phi_{ii} < 0$, and constant return to scale. The planner's problem is similar to before with $\dot{k} = F^m - h_m - c_m - (\delta + \nu) k$ replacing (4), $\sum_{i=1}^m n_i q_i = \Phi$ as an addition resource constraint and $\left\{ h_m, (c_i, q_i)_{i=1,..,m} \right\}$ as additional control variables.

The static efficiency conditions are:

$$\frac{v_i}{v_m} = \frac{F_K^m}{F_N^i} = \frac{F_N^m}{F_N^i} = \frac{F_Q^m}{F_Q^i} = \frac{\Phi_i}{\Phi_m}; \qquad \forall i,$$

which implies $k_i = k$, $q_i = \Phi$, $p_i = A_m / A_i$, $\forall i$, and $y = A_m k^{\alpha} \Phi^{\beta}$. Define $h \equiv \sum_{i=1}^m p_i h_i$. To solve for h, use planner's optimal conditions for h_m and q_m to obtain $1 = \beta \Phi_m A_m k^{\alpha} \Phi^{\beta - 1}$. But Φ is homogenous of degree one: $\Phi = \sum_{i=1}^m \Phi_i h_i = \sum_{i=1}^m \Phi_m p_i h_i = \Phi_m h$, we have $h = \beta y$, together with static efficiency,

$$\dot{k} = A_m k^{\alpha} \Phi^{\beta} \left(1 - \sum\nolimits_{i=1}^{m-1} n_i \right) - h_m - c_m - (\delta + \nu) k = h (1 - \beta) / \beta - c - (\delta + \nu) k.$$

The dynamic efficiency condition is $-\dot{v}_m / v_m = \alpha A_m k^{\alpha - 1} \Phi^{\beta} - (\delta + \rho + \nu)$, so

$$\dot{c}/c = \alpha h / (\beta k) - (\delta + \rho + \nu), \quad \dot{k}/k = (1 - \beta) h / (\beta k) - c/k - (\delta + \nu).$$

Constant $\dot{c}/c$ requires constant h/k. Then constant $\dot{k}/k$ requires constant c/k. Thus, $\dot{h}/h$ must be constant, i.e. Φ/Φ_m must be growing at a constant rate. Suppose Φ is CES function, $\Phi = \left(\sum_{i=1}^m \varphi_i h_i^{(\eta-1)/\eta} \right)^{\eta/(\eta-1)}$, then the static efficiency conditions imply $\forall i$, $p_i h_i / h_m = (\varphi_i / \varphi_m)^{\eta} (A_m / A_i)^{1-\eta} \equiv z_i$, $h = Z h_m$, where $Z \equiv \sum_{i=1}^m z_i$, so $\Phi_m = \varphi_m^{\eta/(\eta-1)} Z^{1/(\eta-1)}$ and $\Phi = \left(\beta A_m k^{\alpha} \varphi_m^{\eta/(\eta-1)} Z^{1/(\eta-1)} \right)^{1/(1-\beta)}$. Hence,

$$\begin{aligned} h &= \Phi/\Phi_m = (\beta A_m k^{\alpha})^{1/(1-\beta)} \left(\varphi_m^{\eta/(\eta-1)} Z^{1/(\eta-1)} \right)^{\beta/(1-\beta)} \\ &\implies (1 - \beta) \dot{h}/h = \left(\gamma_m + \alpha \dot{k}/k \right) + \beta \left(\sum\nolimits_{i=1}^m (z_i / Z) \gamma_i - \gamma_m \right) \end{aligned}$$

which is constant only if $(1 - \eta) \sum_{i=1}^m (z_i/Z) \gamma_i \left(\sum_{i=1}^m (z_i/Z) \gamma_i - \gamma_i \right) = 0$. Given γ are not the same across all i, must have $\eta = 1 : \Phi = \prod_{i=1}^m h_i^{\varphi_i}$, $Z = 1/\varphi_m$, and $z_i = \varphi_i / \varphi_m$, $\forall i$. The static efficiency conditions imply $\Phi = h_m \prod_{i=1}^m (z_i A_i / A_m)^{\varphi_i}$ and so $\Phi_m = \varphi_m \Phi / h_m = \prod_{i=1}^{m} (\varphi_i A_i / A_m)^{\varphi_i}$. But $\Phi = [\beta A_m k^\alpha \Phi_m]^{1/(1-\beta)}$, so $h = \Phi/\Phi_m = (\beta A_m k^\alpha)^{1/(1-\beta)} \Phi_m^{\beta/(1-\beta)}$. The dynamic equations become:

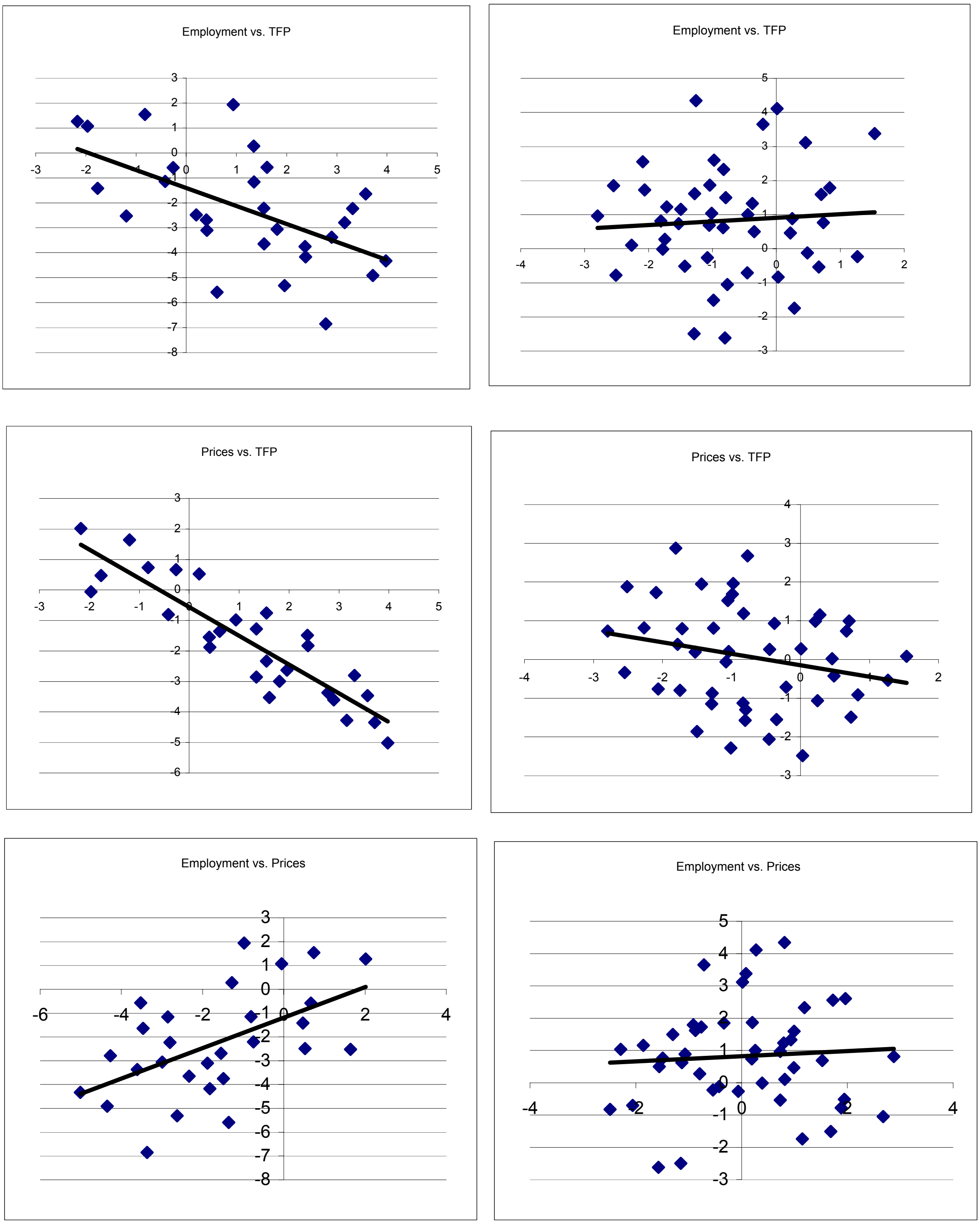
$$
\begin{aligned}
\frac{\dot{c}}{c} + \delta + \rho + \nu &= \frac{\alpha}{\beta k} \left[(\beta A_m k^\alpha) \Phi_m^\beta \right]^{\frac{1}{1-\beta}} = \alpha \left[k^{\alpha+\beta-1} A_m (\beta \Phi_m)^\beta \right]^{\frac{1}{1-\beta}} , \\
\frac{\dot{k}}{k} + \frac{c}{k} + \delta + \nu &= \frac{(1-\beta)}{\beta k} \left[(\beta A_m k^\alpha) \Phi_m^\beta \right]^{\frac{1}{1-\beta}} = (1-\beta) \left[k^{\alpha+\beta-1} A_m (\beta \Phi_m)^\beta \right]^{\frac{1}{1-\beta}} .
\end{aligned}
$$

Define $c_e \equiv c A^{-(1-\beta)/(1-\alpha-\beta)}$ and $k_e \equiv k A^{-(1-\beta)/(1-\alpha-\beta)}$, where $A \equiv \left[A_m (\beta \Phi_m)^\beta \right]^{1/(1-\beta)}$, and $\gamma \equiv \dot{A}/A = \left[\gamma_m + \beta \sum_{i=1}^{m} \varphi_i (\gamma_i - \gamma_m) \right] / (1-\beta) = \gamma_m + \left(\beta \sum_{i=1}^{m} \varphi_i \gamma_i \right) / (1-\beta)$,

$$
\begin{aligned}
\dot{c}_e / c_e &= \alpha k_e^{(\alpha+\beta-1)/(1-\beta)} - [\delta + \rho + \nu + (1-\beta) \gamma / (1-\alpha-\beta)] , \\
\dot{k}_e / k_e &= (1-\beta) k_e^{(\alpha+\beta-1)/(1-\beta)} - c_e/k_e - [\delta + \nu + (1-\beta) \gamma / (1-\alpha-\beta)] ,
\end{aligned}
$$

which imply existence and uniqueness of a balanced growth path. $\forall i = 1, ..m-1$, obtain n_i using $F^i = c_i + h_i$, i.e. $A_i n_i k^\alpha \Phi^\beta p_i = p_i (c_i + h_i) = x_i c_m + z_i h_m = c x_i / X + \varphi_i h$. Substitute p_i and h to obtain $n_i y = c x_i / X + \varphi_i \beta y$, finally

$$
n_i = (c/y)(x_i/X) + \varphi_i \beta; \quad \forall i, \qquad n_m = [(c/y)(x_m/X) + \varphi_m \beta] + [1 - \beta - c/y] .
$$

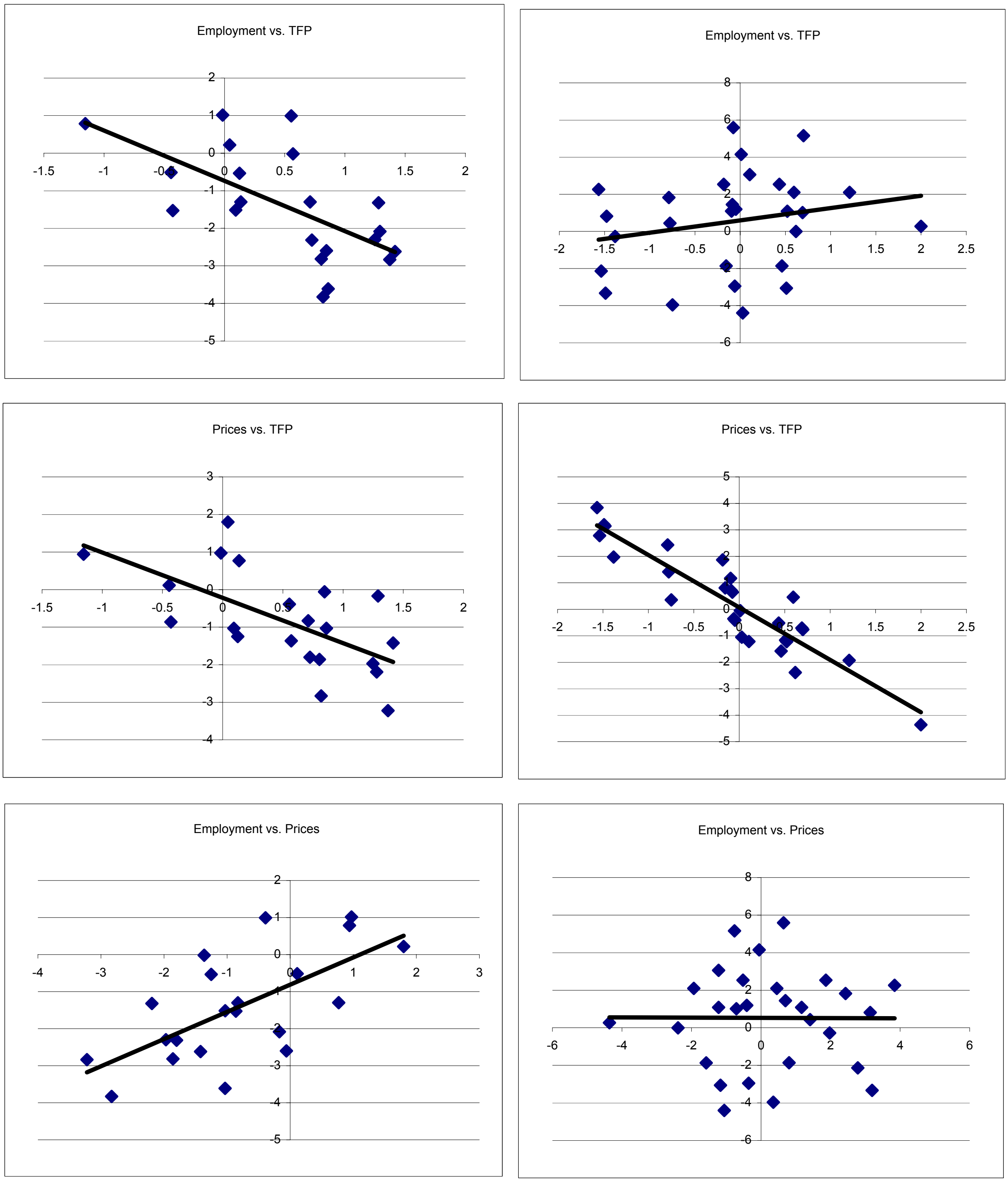


(a) "Consumption" sectors

(b) "Manufacturing" sectors

Figure 1

Changes in relative employment shares, relative TFP, and relative prices.
(percent, United States, averages for 1970-1993)



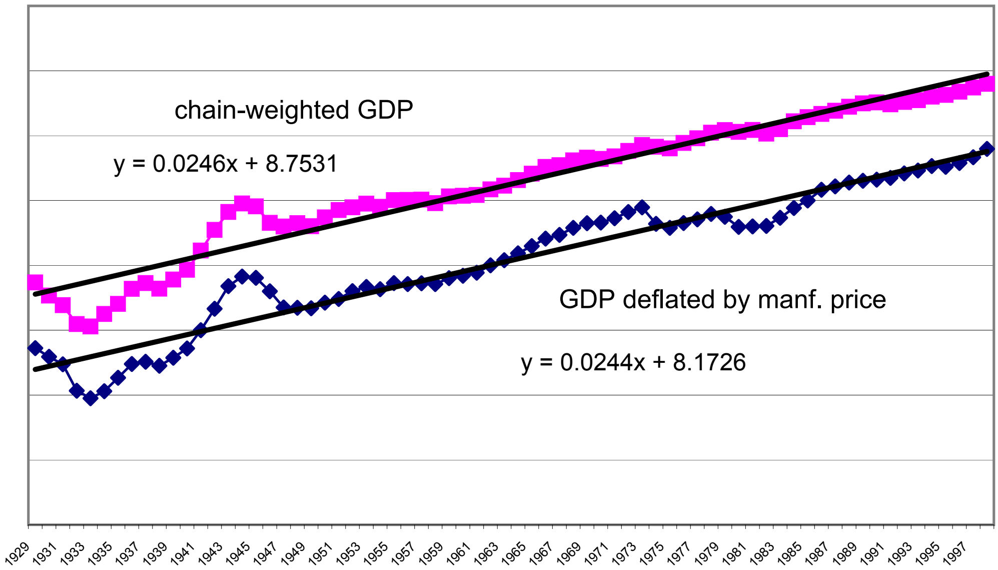
(a) "Consumption" sectors

(b) "Manufacturing sectors

Figure 2

Changes in relative employment shares, relative TFP, and relative prices
(percent, United Kingdom, averages for 1970-1990)

Figure 3
Chain-weighted GDP and GDP deflated by manufacturing price, US, per capita, log scale
(base years differ)



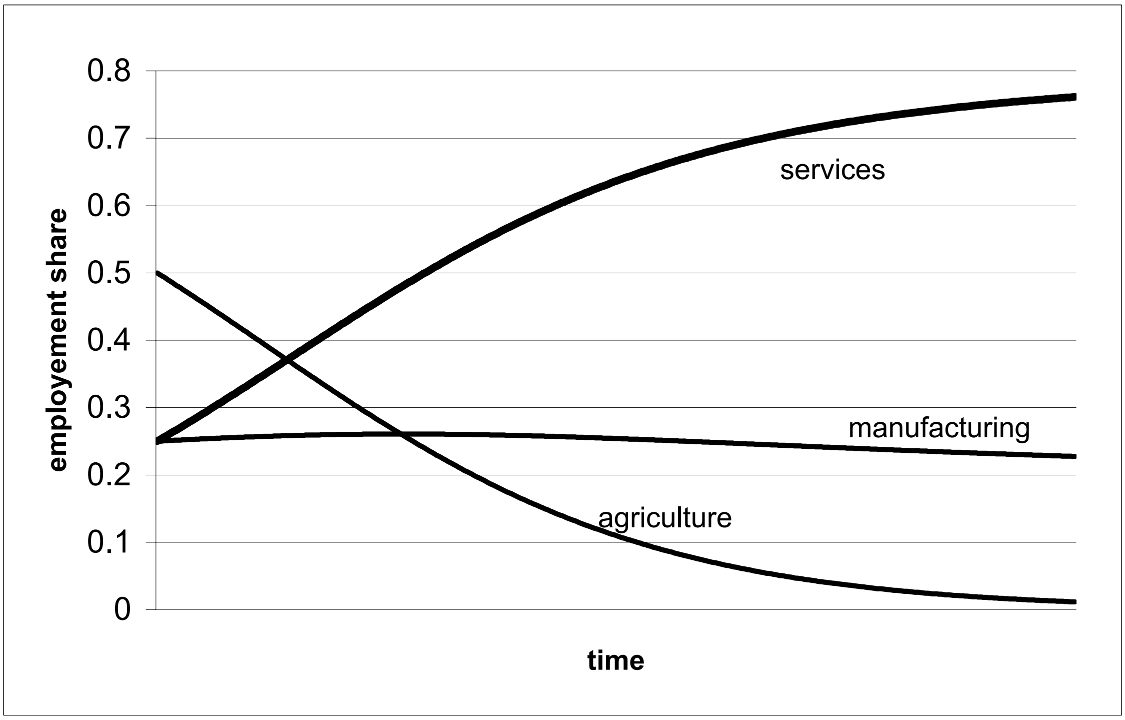
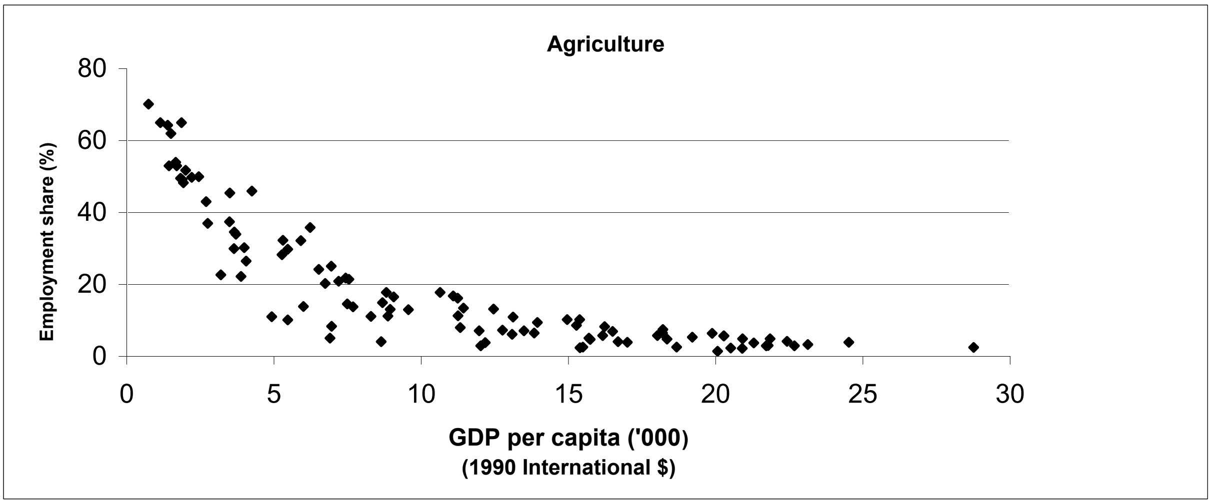
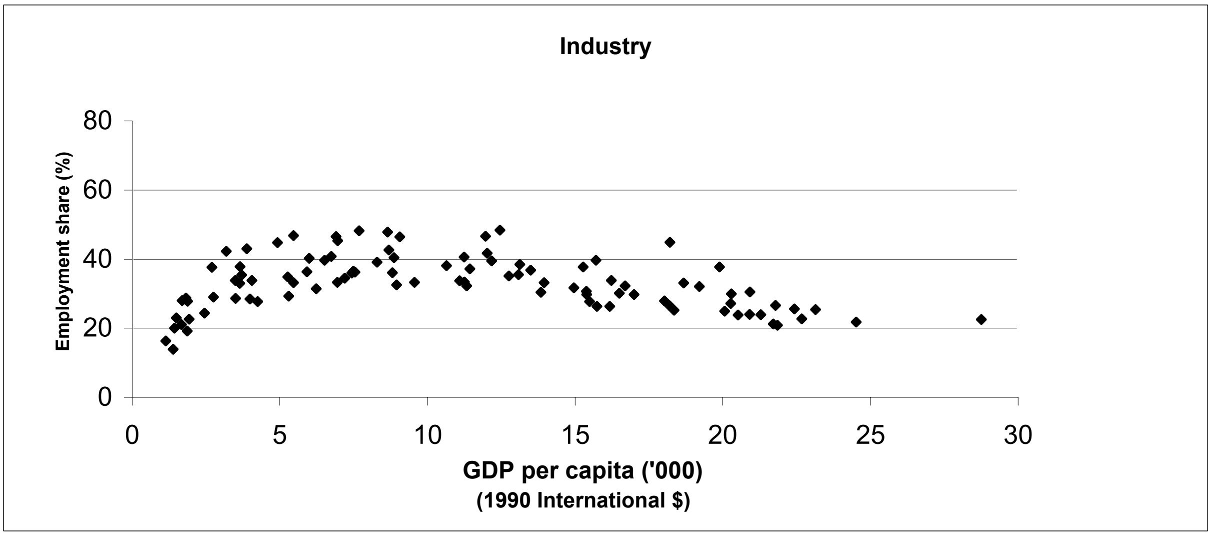


Figure 4

Structural transformation in a three-sector economy





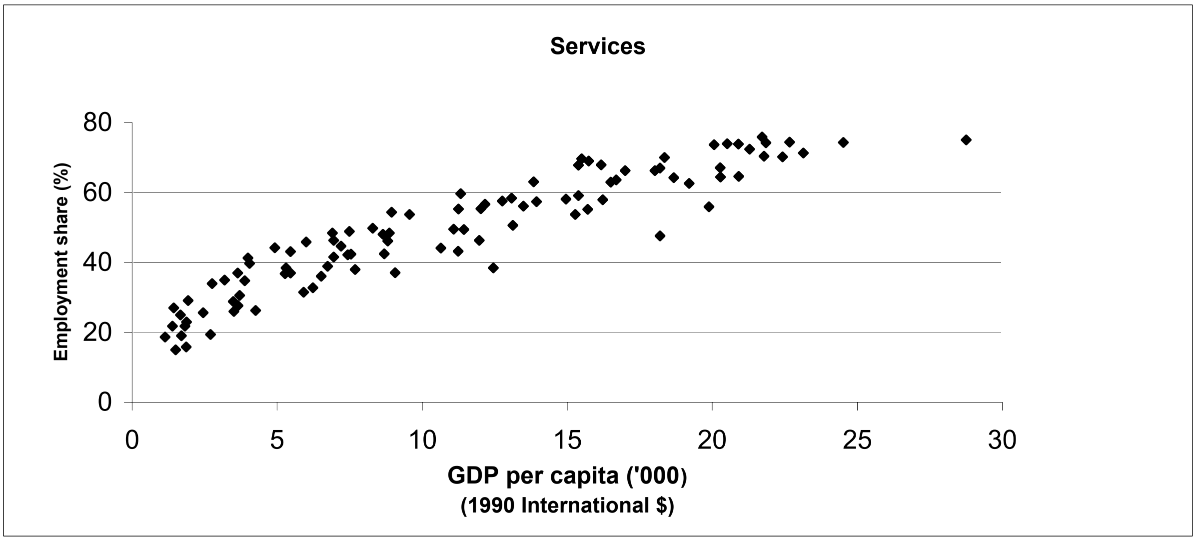
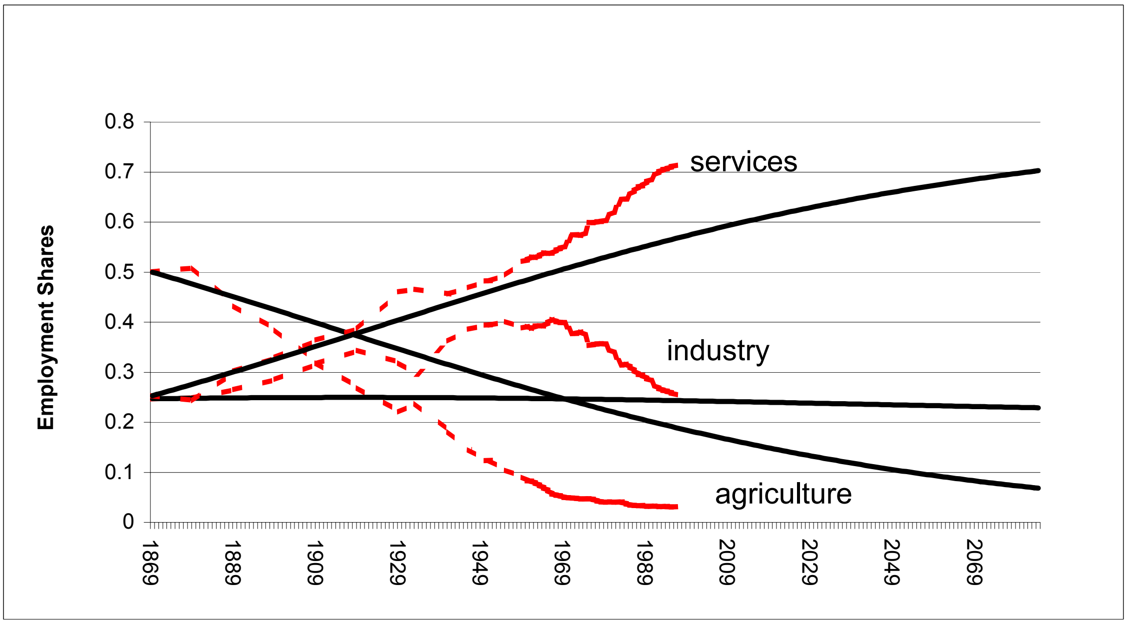
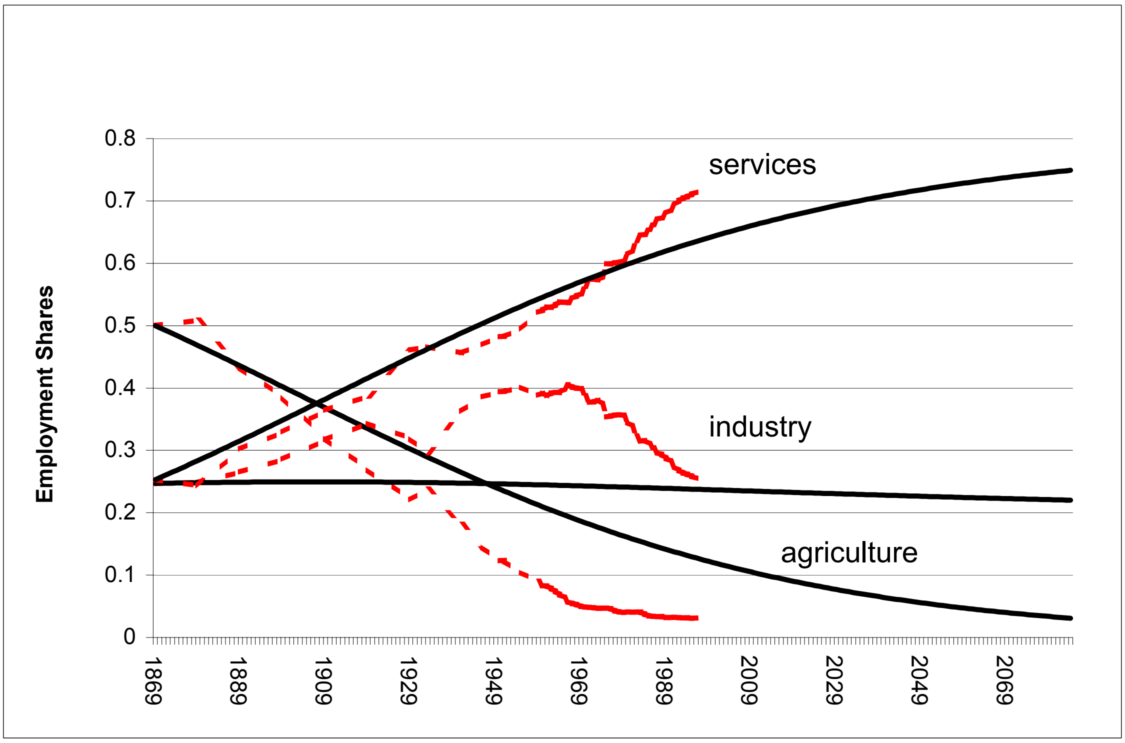


Figure 5

Sectoral Employment Shares 1870-2001
(Sixteen OECD countries and seven years)



(a) epsilon = 0.3



(b) epsilon = 0.1

Figure 6

Structural transformation in the US economy
___________ model calibration …………...data